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11. SUPPLEMENTARY NOTES

The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.

12 a. DISTRIBUTION / AVAILABILITY STATEMENT

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12 b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

The main goal of this project is to develop a systematic mathematical theory for the robust real time feedback control and stochastic analysis of unsteady transonic aerodynamics of helicopter rotor blades, supersonic ballistic projectiles and propagation of blast waves in the presents of adverse external disturbances. The proposed work builds upon similar scientific advances by the principal investigator in the context of incompressible fluid dynamics, magneto-hydrodynamics and combustion models during the past two decades under ONR, DARPA and ARO sponsorship. The gas dynamic models to be used in the research will be either the Euler equations or the quasilinear unsteady potential equations. These nonlinear hyperbolic or elliptic-hyperbolic type mixed equations are subjected to additive and multiplicative external disturbances modeled as Gaussian, Poisson and Levy type noise forces. Main results to be expected are mathematical characterization of entropy solutions for stochastic hyperbolic systems of conservation laws, state estimation and feedback control analysis as well as assessment of the impact of noise in controlled and uncontrolled aerodynamic flows of the above type. The outcomes of this research will be expected to impact a number of other subjects of importance to the Army such as combustion control and multiphase fluid dynamics. Random free stream velocities and boundary conditions will be incorporated in to the deterministic aerodynamic code developed by the principal investigator for supersonic cones at angle of attack and also in the a new code to be developed for the stochastic transonic small disturbance equation to gain further insights in to the theoretical work. A unique aspect of this research is the close interactions and connections the principal

14. SUBJECT TERMS

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Enclosure 1

REPORT DOCUMENTATION PAGE (SF298)
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1. Utpal Manna, J. L. Menaldi and S. S. Sritharan, “Stochastic Tidal Dynamics Equation”, in Infinite Dimensional Stochastic Analysis, Edited by A. N. Sengupta and P. Sundar, World Scientific Publishers, 2008.

2. Jingling Guan and S. S. Sritharan, “A Problem of Hyperbolic-Elliptic Type Conservation Laws on Manifolds that Arises in Delta Wing Aerodynamics”, International Journal of Contemporary Mathematical Sciences, Vol. 3, 2008, no.15, 721-737.

3. Utpal Manna, J. L. Menaldi and S. S. Sritharan, “Stochastic Navier-Stokes Equations with Artificial Compressibility”, Communications on Stochastic Analysis, Vol.1, No.1, (2007), 123-139.

4. Utpal Manna and S. S. Sritharan, “Lyapunov Functionals and Local Dissipativity for the Vorticity Equation in L^p and Besov spaces”, Differential and Integral Equations, Vol. 20, No. 5, (2007), 581-598.

5. Utpal Manna, P. Sundar and S. S. Sritharan, “Large Deviations for Stochastic Turbulent Shell Models”, Submitted for publications in NoDEA (Nonlinear Differential Equations and Applications), 2008.

6. K. Sakthivel, K. Balachandran and S. S. Sritharan, “Controllability of Parabolic Systems Arising in Combustion of Material with Memory”, Submitted for publication in SIAM Journal of Control and Optimization, 2008.

7. S. Popa, S. and Sritharan S.S. (2008), Nonlinear Filtering of Itˆo-L´evy Stochastic Differential Equations with Continuous Observations, submitted to Communications on Stochastic Analysis.

8. Utpal Manna, Barbara Rudiger and S. S. Sritharan, “Stochastic 2-D Navier-Stokes equations of jump type: existence, uniqueness and large deviations”, submitted for publication.

C. Final Progress Report

(4) Statement of the Problem Studied:

The purpose of this research is to bring rigorous stochastic estimation and control methodologies to impact certain Army' priority technological needs, namely, the task of achieving agile aerodynamic design of helicopter blades, management of supersonic flow past ballistic projectiles and the propagation of blast waves in random environment. The proposed work will be a very novel combination supersonic-transonic aerodynamic design with stochastic analysis and control theory of fluid dynamics, building upon various advances by the principal investigator since the 1980s under DoD sponsorship. The specific mathematical goal will be to perform theoretical and computational study of fully nonlinear hyperbolic and mixed elliptic-hyperbolic type partial differential equations that arise in these contexts subjected to adverse noise and to design feedback controllers for agile performance. Specific deliverables are:

1. **Optimal Control of Entropy Solutions:** Characterization of stochastic entropy solutions for nonlinear hyperbolic system of conservation laws and second order quasilinear hyperbolic and mixed partial differential equations that arise in blast wave dynamics, supersonic projectile aerodynamics and helicopter blade aerodynamics subjected to a variety of noise disturbances and control forcings.

2. **Nonlinear Filtering, Dynamic Programming and Impulse Control:** State estimation, feedback synthesis and control analysis for the real-time management of supersonic and transonic flow with bow and body-embedded shock waves in the presense of noise.

3. **Stochastic Stability & Large Deviation Analysis:** A mathematical assessment of the impact of noise and stochastic stability in supersonic and transonic flow fields addressing issues such as shock attachment-detachment, shock-free supercritical flows over propeller blades and blast waves.

4. **Computational Experiments:** Stochastic boundary conditions and free stream conditions will be introduced in to an existing computer code for supersonic conical flows and a new code to be developed for transonic small disturbace equation to throw more light in to the stochastic analysis and control theoretical aspects of supersonic and transonic aerodynamics.

(5) Summary of Most Important Results:

This research has been well centered in doctoral student training and a total of five doctoral students (named below) all of whom have taken many of the principal investigator's advanced graduate level research courses such as stochastic processes, control theory of Markov processes, advanced functional analysis, advanced partial differential equations as well as mathematical theory of finance. This has created a unique climate at University of Wyoming mathematics department with well trained doctoral students. In fact during the past week two of the students attended Professor Nualart's ten-lecture CBMS lecture series at Kent State University where they further sharpened their background in stochastic calculus with Malliavin calculus and supporting seminars in that subject. We describe below the exciting research performed by these doctoral students who are supported by this ARO project. The principal investigator has discussed the theory of semimartingales and jump processes extensively in his classes. Likewise he has also taught the students techniques of nonlinear partial differential equations as well as mathematical theory of conservation laws and shock waves as applied to compressible fluid dynamics.

The main progress of this research project has been in identifying the right mathematical framework to incorporate stochasticity in compressible inviscid fluid dynamics so that we can develop control and filtering theory to these models and also study large deviation property to assess the impact of noise. It turns out that for multi-dimensional scalar conservation laws we have the well known

Kruzkov-Crandall theory for the deterministic situation (for entropy solutions) which ties the subject very closely to nonlinear semigroup theory in the L^1-space since the nonlinear operator in this case is m-accretive in this space. Hence the right mathematical model then follows by starting with nonlinear deterministic evolution equations governed by nonlinear m-accretive operators and the noise can be an additive term incorporating additive and multiplicative noise cases. We have in this way found a way to incorporate by continuous and jump noise cases by working with Hilbert space-valued Levy process. The control and filtering developments then follow generalizing the previous work of the principal investigator. Mr. Saikat Mukherjee is pursuing his PhD thesis along these lines and the first result we have (along with the solvability theorem for controlled stochastic conservation law) is the existence of optimal controls. We are currently working of generalizing the result to multi-dimensional system of conservation laws. The theory of entropy solutions for system of conservation laws is incomplete even for the deterministic case so the first step will focus on 1-dimensional systems in the presence of noise. Nonlinear filtering of discontinuous stochastic processes using discontinuous measurements is another component of this research. Ms. Sylvia Popa is pursuing her research in this subject and her preliminary results are for finite dimensional processes and even in this case the results are new. The results so far are careful derivation of the Fujisaki-Kallianpur-Kunita and Zakai equations for the special case of continuous measurements where the generator of the signal Markov process is a Levy generator. Uniqueness theorems for evolution of filter have been proven. We will soon generalize this to infinite dimensional processes of the type that occur in the specific applications addressed in this project. In the finite dimensional context this particular result has a number of financial applications which are also been investigated to gain further insights. The case of discontinuous measurements is significantly more challenging since due to the non-uniqueness of Girsanov transform which is a crucial intermediate step. This particular component of research is very important in helicopter aerodynamic diagnostics which involve such abruptly changing noise structure in the fluid dynamic part as well as observation. Mr. B. P. W. Fernando is another doctoral student focusing on impulse and stopping time problems for infinite dimensional semi-martingales that arise in stochastic conservation laws of fluid dynamics. He has been generalizing a corresponding theory by J. L. Menaldi and the principal investigator for the case of incompressible fluid dynamics. We have identified crucial steps in the previous theory which need generalization and making good progress by focusing on a general class of infinite dimensional semi-martingales that arise in fluid dynamics and addressing the variational and quasi-variational inequalities that arise in this context. Meng Xu is generalizing some initial results on Malliavin calculus available for fluid dynamics models (due to S. R. S. Varathan and also Pardoux-Mattingly) to two dimensional incompressible Navier-Stokes equations with the hope of proving a large variety of results including null controllability, convergence of approximation schemes etc. Initial direction is in Gaussian noise but we will be also looking at Levy noise.

(6) Bibliography

1. Utpal Manna, J. L. Menaldi and S. S. Sritharan, “Stochastic Tidal Dynamics Equation”, in **Infinite Dimensional Stochastic Analysis**, Edited by A. N. Sengupta and P. Sundar, World Scientific Publishers, 2008.

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